An N-Point Linear Solver for Line and Motion Estimation with Event Cameras

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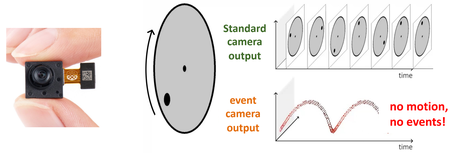
Motivation

Use events generated by a 3D line and gyro. readings from IMU to recover **partial linear velocity and line parameters** with a fast and robust linear solver. Applying a novel velocity averaging scheme, we fuse these partial observations to obtain full linear camera velocity.

Contributions

1. A linear solver for minimal and overdetermined (N>=5 events) systems, that is 600x faster than polynomial solvers.
2. A 3 DoF angle-axis-based line parametrization that improves the numerical stability of existing solvers.
3. A full characterization of degeneracies and solutions of the solver, and manifolds spanned by the events.
4. A geometry-inspired velocity averaging scheme that is simpler and faster than existing method.

What is an event camera?



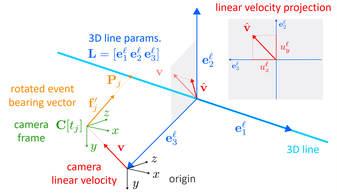
- Measures a stream of **asynchronous brightness changes ("events")**
- Advantages: high temporal resolution, reduced motion blur, low power consumption, high pixel bandwidth, high dynamic range

Methodology:

Events from temporal slice → **Extract manifold** → **Average partial velocity results** → **Camera velocity**

Linear Solver

- Stack linear incidence relations, one for each event
- Solve for line and velocity unknowns using SVD
- Reconstruct the unknowns using simple vector algebra



Incidence Relationship

$$\underbrace{e_1^{\ell T}\left((R_\ell u_\ell t_j')\times f_j'\right)}_{\bar{m}_j} + \underbrace{\left(-e_2^{\ell}\right)^T f_j'}_{\bar{d}_j} = 0 \quad \text{simplify} \Rightarrow \quad t_j' f_j'^T\left(u_z^\ell e_2^\ell - u_y^\ell e_3^\ell\right) + f_j'^T e_2^\ell = 0$$

unknowns; measurements

stack $\Rightarrow$ $\underbrace{\begin{bmatrix} t_1' f_1'^T & f_1'^T \\ \vdots & \vdots \\ t_N' f_N'^T & f_N'^T \end{bmatrix}}_{A\in\mathbb{R}^{N\times 6}} \begin{bmatrix} u_z^\ell e_2^\ell - u_y^\ell e_3^\ell \\ e_2^\ell \end{bmatrix} = 0$

last column of V from SVD(A) $\Rightarrow$ $\pm\hat{\mathbf{x}} = \begin{bmatrix} \hat{\mathbf{x}}_{1:3} \\ \hat{\mathbf{x}}_{4:6} \end{bmatrix} = \begin{bmatrix} u_z^\ell e_2^\ell - u_y^\ell e_3^\ell \\ e_2^\ell \end{bmatrix}$

multiple solutions

reconstruct unknowns $\Rightarrow$

$$e_2^\ell = \hat{\mathbf{x}}_{4:6}$$
$$u_z^\ell = \hat{\mathbf{x}}_{1:3}^T \hat{\mathbf{x}}_{4:6}$$
$$e_1^\ell = \pm\frac{\hat{\mathbf{x}}_{1:3}\times\hat{\mathbf{x}}_{4:6}}{\|\hat{\mathbf{x}}_{1:3}\times\hat{\mathbf{x}}_{4:6}\|}$$
$$u_y^\ell = \pm\|\hat{\mathbf{x}}_{1:3}\times\hat{\mathbf{x}}_{4:6}\|$$

multiple solutions

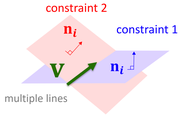
collect variables $\Rightarrow$

Solution

$$\boldsymbol{R}_\ell = \begin{bmatrix} e_1^\ell & e_2^\ell & e_3^\ell \end{bmatrix} \rightarrow \boldsymbol{\theta} = (\log \mathbf{R}_\ell)^\vee$$
$$\boldsymbol{u}_\ell = \begin{bmatrix} 0 & u_y^\ell & u_z^\ell \end{bmatrix}$$

Velocity Averaging Scheme

- Stack linear velocity constraint, one for each line
- Solve for full velocity using SVD



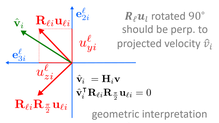
Partial Velocity Measurements

Unobservable due to aperture problem; line index

$$\boldsymbol{v} = u_{x\ell}^\ell e_{1\ell}^\ell + u_{y\ell}^\ell e_{2\ell}^\ell + u_{z\ell}^\ell e_{3\ell}^\ell$$

geometric constraint $\Rightarrow$

$$\boldsymbol{v}^T\left(e_{1\ell}^\ell \times \boldsymbol{v}\right) = \boldsymbol{v}^T\left(u_{y\ell}^\ell e_{3\ell}^\ell - u_{z\ell}^\ell e_{2\ell}^\ell\right) = 0$$



stack $\Rightarrow$ $\underbrace{\begin{bmatrix} u_{x1}^\ell e_{21}^\ell - u_{y1}^\ell e_{31}^\ell \\ \vdots \\ u_{xM}^\ell e_{2M}^\ell - u_{yM}^\ell e_{3M}^\ell \end{bmatrix}}_{A\in\mathbb{R}^{N\times 3}} \boldsymbol{v} = 0$

last column of V from SVD(A) $\Rightarrow$

Solution

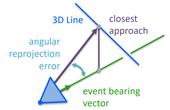
$$\boldsymbol{v} = \pm\hat{\mathbf{x}}$$

Real World Experiments

error in degrees; linear solver on inlier events after GC-RANSAC; … after non-linear optimization

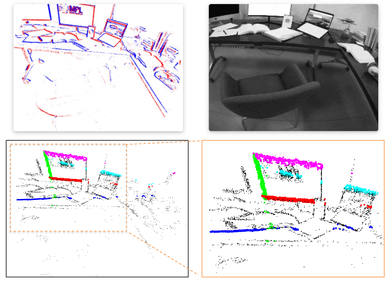
Seq. Name	CELC+opt [Peng BMVC'21]	Gröbner [Gao ICCV'23]	Linear only	Linear w/ non-min. solver	Linear w/ non-lin opt.
mountain-normal	27.7/29.3	33.5/33.6	25.2/21.4	17.0/17.2	**16.5/14.6**
desk-normal	26.6/26.6	26.4/26.7	22.7/23.4	**19.8/19.2**	22.1/20.7
sofa-normal	24.0/26.1	31.4/29.5	21.9/17.6	20.6/16.1	**19.9/15.0**

- Evaluation on real-world data sequences from the VECtor Benchmark
- Addition of robust geometric model fitting method **GC-RANSAC** to reject spurious events, and trigger local refinement for early termination
- Non-linear optimization step minimizes the **angular reprojection error**

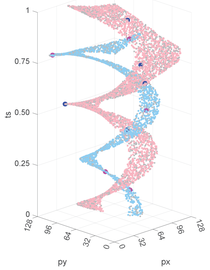


Snapshots of Clustering Results

Real World



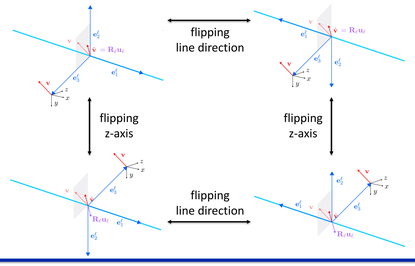
Simulation



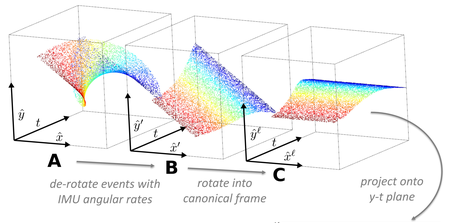
Each color indicates a cluster of events generated by single 3D line

Multiple Solutions

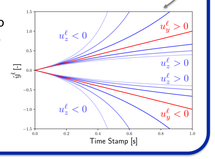
- The proposed solver returns up to 4 different solutions
- One symmetry corresponds to **flipping along the z-axis**
- The second corresponds to **flipping the line direction**
- ***Disambiguate by checking the line is "in front of the camera"***



Characterizing Event Manifolds

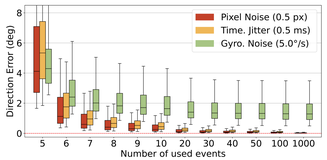


- All event manifolds can be mapped into a **canonical** representation in frame $\boldsymbol{R}_\ell$
- In this frame, the manifold is only parametrized by u_z^ℓ, u_y^ℓ
- u_z^ℓ controls the curvature, while u_y^ℓ controls the slope

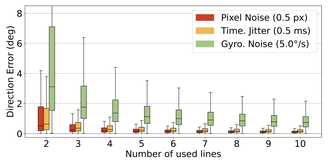


Quantitative Results

Our linear solver and velocity averaging scheme can be extended to **an arbitrary number of events/lines**.



- As few as **10 events** are sufficient to substantially reduce the error from noisy measurements.



- As few as **5 lines** significantly reduce the error induced by noisy measurements.

Method	Runtime [µs] min.	Runtime [µs] Avg.	Error Rate (%) >0.1º	Error Rate (%) >1.0º
Gröbner [Gao ICCV '23]	1,893	2,046	1.00	0.28
Linear (ours)	**3**	**3.25**	**0.00**	**0.00**

directional error over velocity

Linear solver is **about 600 times faster** than the Gröbner basis solver.

mean/median; error in degrees

Method	num. events	Pixel Noise (0.5 px)	Time Jitter (0.5 ms)	Gyro. Noise (5.0º/s)
Gröbner [Gao ICCV'23]	5	7.80/1.67	3.61/0.83	7.48/3.09
Linear (ours)	5	5.53/1.24	2.87/0.73	6.53/2.47
Linear (ours)	**10**	**0.46/0.15**	**0.17/0.12**	**1.50/1.17**

Outperforms the Gröbner basis solver with 5 events. **Much more accurate with 10 events.**

Project Webpage

https://mgaoling.github.io/eventail/

Sponsors

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